

INSTITUTE AND FACULTY OF ACTUARIES

Curriculum 2019

SPECIMEN EXAMINATION

Subject CM1A – Actuarial Mathematics

Time allowed: Three hours and fifteen minutes

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 16 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

- 1** A 91-day treasury bill is issued by the government at a simple rate of discount of 8% per annum.
- Calculate the annual effective rate of return obtained by an investor who purchases the bill at issue. [3]
CT1 September 2011 Q1
- 2** Describe three methodologies for adding regular reversionary bonuses to the sum assured of a conventional with profits assurance. [3]
- 3** Calculate ${}_{0.5}q_{75.25}$ using the assumption of a constant force of mortality.
- Basis:
Mortality AM92 [3]
CT5 September 2011 Q2
- 4** Explain what a stochastic model is and how it differs from a deterministic model. [4]
CT4 April 2014 Q3
- 5** Calculate:
- (a) ${}_{20|}q_{[40]}$
(b) ${}_{10}P_{[70]+1}$
(c) $\ddot{a}_{[40]:\overline{20}|}^{(12)}$
- Basis:
Mortality AM92
Rate of interest 6% per annum [4]
- 6** A three-year unit-linked endowment assurance policy is sold to a male life aged 40 exact. The profit signature for this policy, calculated using AM92 Select mortality and making no allowance for surrenders, is:
- (−209.80, 253.55, 109.85)
- It is now assumed for the cash flows for this policy that 15% of all policies in force at the end of the first policy year are surrendered at that time. The surrender value payable at that time is the bid value of units at the end of the policy year less a surrender penalty of £500. There are no other changes to the policy.
- (a) Calculate the revised profit signature in the first policy year.
(b) Comment on the impact on the profit signature in the second and third policy years. [4]
CT5 September 2013 Q15

- 7 Calculate the single premium payable for a temporary reversionary annuity of £12,000 per annum payable monthly in arrear to a female life currently aged 55 exact on the death of a male life currently aged 50 exact. No payment is made after 20 years from the date of purchase.

Basis:

Rate of interest	4% per annum	
Mortality of male life	PMA92C20	
Mortality of female life	PFA92C20	
Expenses	Nil	[4]

CT5 September 2010 Q3

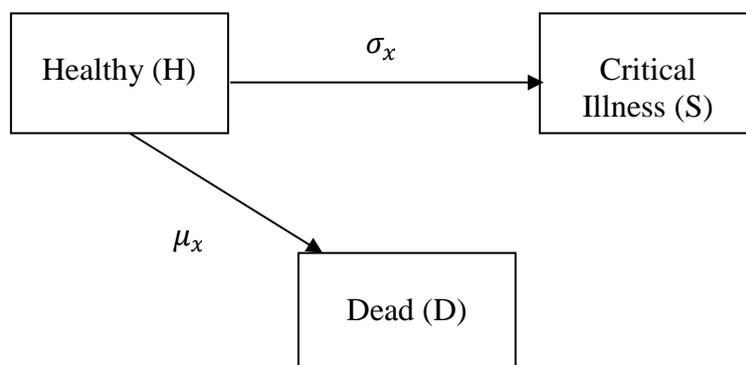
- 8 A life insurance company issues a 35-year endowment assurance policy to a life aged 30 exact. The sum assured of 25,000 is payable on survival to the end of the term or at the end of the year of death if earlier. Level annual premiums are payable in advance throughout the term of the policy or until earlier death.

In Country A the pricing basis used is an interest rate of 4% per annum and mortality of AM92 Ultimate. In country B the pricing basis used is an interest rate of 3.5% per annum and an assumption of no mortality.

- (i) Calculate the annual premiums charged in Country A and in Country B. Ignore any expenses. [2]

- (ii) Comment on your results in part (i). [2]
- [Total 4]

- 9 A life insurance company issues a 10-year critical illness policy which provides a benefit of 100,000 on death or earlier diagnosis of a critical illness. The transition diagram for this policy is given by



Calculate the expected present value of the policy under the following basis:

$\sigma_x = 0.04$ for all ages

$\mu_x = 0.01$ for all ages

force of interest, $\delta = 5\%$ per annum

[6]

Based on CT5 September 2015 Q7

- 10** (i) List eight factors which should be considered when assessing whether a model is suitable for a particular application. [4]
- (ii) State, giving reasons, a factor which would be particularly important in each of the following applications:
- Calculating the pension contribution for a medium sized pension scheme.
 - Helping a friend construct a business case to secure a loan from a bank for his new ice-cream van venture.
 - Working out how much it will cost to buy each member of your team their favourite cake on your birthday in six months' time. [3]

[Total 7]

CT4 September 2014 Q2

- 11** In January 2014, the government of a country issued an index-linked bond with a term of two years. Coupons were payable half-yearly in arrear, and the annual nominal coupon rate was 4%. Interest and capital payments were indexed by reference to the value of an inflation index with a time lag of six months.

A tax-exempt investor purchased £100,000 nominal at issue and held it to redemption. The issue price was £98 per £100 nominal.

The inflation index was as follows:

<i>Date</i>	<i>Inflation Index</i>
July 2013	110.5
January 2014	112.1
July 2014	115.7
January 2015	119.1
July 2015	123.2

- (i) Calculate the investor's cashflows from this investment and state the month when each cashflow occurs. [3]
- (ii) Calculate the annual effective money yield obtained by the investor to the nearest 0.1% per annum. [3]

[Total 6]

CT1 April 2010 Q2 (dates changed)

- 12** An insurance company has liabilities of £6 million due in exactly 8 years' time and a further £11 million due in exactly 15 years' time.

The assets held by the insurance company consist of:

- a 5-year zero-coupon bond of nominal amount £5.5088 million; and
- a 20-year zero-coupon bond of nominal amount £13.7969 million.

The current rate of interest is 8% per annum effective at all durations.

- (i) Show that the first two conditions of Redington's theory for immunisation against small changes in the rate of interest are satisfied. [5]
- (ii) Explain, without doing any further calculations, whether the insurance company will be immunised against small changes in the rate of interest. [2]
- [Total 7]

Based on CT1 April 2013 Q7

- 13** An individual can obtain a force of interest per annum at time t , measured in years, as given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.01t & 0 \leq t < 4 \\ 0.07 & 4 \leq t < 6 \\ 0.09 & 6 \leq t \end{cases}$$

- (i) Calculate the amount the individual would need to invest at time $t = 0$ in order to receive a continuous payment stream of \$3,000 per annum from time $t = 4$ to $t = 10$. [6]
- (ii) Calculate the equivalent constant annual effective rate of interest earned by the individual in part (i). [3]
- [Total 9]

CT1 April 2014 Q11

14 A fixed-interest bond pays annual coupons of 5% per annum in arrear on 1 March each year and is redeemed at par on 1 March 2025.

On 1 March 2007, immediately after the payment of the coupon then due, the gross redemption yield was 3.158% per annum effective.

(i) Calculate the price of the bond per £100 nominal on 1 March 2007. [3]

On 1 March 2012, immediately after the payment of the coupon then due, the gross redemption yield on the bond was 5% per annum.

(ii) State the new price of the bond per £100 nominal on 1 March 2012. [1]

A tax-free investor purchased the bond on 1 March 2007, immediately after payment of the coupon then due, and sold the bond on 1 March 2012, immediately after payment of the coupon then due.

(iii) Calculate the gross annual rate of return achieved by the investor over this period. [2]

(iv) Explain, without doing any further calculations, how your answer to part (iii) would change if the bond were due to be redeemed on 1 March 2035 (rather than 1 March 2025). You may assume that the gross redemption yield at both the date of purchase and the date of sale remains the same as in parts (i) and (ii) above. [3]

[Total 9]

CT1 April 2012 Q6

15 On 1 January 2005, a life insurance company issued 1,000 10-year term assurance policies to lives aged 55 exact. For each policy, the sum assured is £50,000 for the first five years and £25,000 thereafter. The sum assured is payable immediately on death and level annual premiums are payable in advance throughout the term of this policy or until earlier death.

The company uses the following basis for calculating premiums and reserves:

Mortality	AM92 Select
Interest	4% per annum
Expenses	Nil

(i) Calculate the gross premium retrospective reserve per policy as at 31 December 2009. [6]

(ii) (a) Give an explanation of your numerical answer to (i) above.

(b) Describe the main disadvantage to the insurance company of issuing this policy.

(c) Give an example of how the terms of the policy could be altered so as to remove this disadvantage.

[3]

There were, in total, 20 deaths during the years 2005 to 2008 inclusive and a further 8 deaths in 2009.

- (iii) Calculate the total mortality profit or loss to the company during 2009. [3]
[Total 12]

CT5 April 2010 Q12 (minor wording changes)

- 16** A life insurance company issues a 15-year increasing term assurance policy to a life aged 50 exact.

The death benefit on the policy, payable immediately on death, is given by the formula:

$$£10,000 \times [6+t] \quad t = 0, 1, 2, \dots, 14$$

where t denotes the curtate duration in years since the inception of the policy.

Level premiums on the policy are payable annually in advance for the term of the policy, ceasing on death if earlier.

Premium Basis

Mortality AM92 Select

Interest 6% per annum

Expenses Initial

£225

Renewal

£65 per annum inflating at 1.92308% per annum, at the start of the second and subsequent policy years (note that the amount quoted is at outset and the increases due to inflation start immediately)

Claim

£275 on termination

- (i) Give an expression for the gross future loss random variable under the policy at the outset. [4]
- (ii) Calculate the annual premium for the policy. [7]
- (iii) Calculate the gross prospective reserve for the policy at the end of the 14th policy year using the elements of the premium basis that are relevant. [4]

[Total 15]

Based on CT5 September 2013 Q23

END OF PAPER