INSTITUTE AND FACULTY OF ACTUARIES

Curriculum 2019

SPECIMEN EXAMINATION

Subject CM2A – Financial Engineering and Loss Reserving

Time allowed: Three hours and fifteen minutes

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 10 questions, beginning your answer to each question on a new page.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is NOT required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 In a market where the CAPM holds there are five risky assets with the following attributes per year.

Asset number	*1*	*2*	*3*	*4*	*5*
Expected return	6%	5%	8%	13%	11%
Market capitalisation (in $)	2.6m	3.9m	5.2m		1.3m
Beta				1.5	

The risk-free rate is $r = 1\%$ p.a.

(i) Calculate the expected return on the market portfolio. [1]

(ii) Determine the market capitalisation of asset 4 and the betas of all the other assets. [3]

(iii) Calculate the beta of a portfolio P which is equally weighted in the five assets and the risk-free asset. [1]

(iv) Explain whether or not this portfolio P lies on the Capital Market Line. [2]
[Total 7]

CT8 April 2012 Q2 (minor wording changes)

2 (i) State the three main assumptions of Modern Portfolio Theory. [3]

Assume Modern Portfolio Theory holds true.

(ii) Write down equations for the expected return, E and variance, V of a portfolio of N securities, defining any notation used. [3]

(iii) Describe how an efficient portfolio can be found. [4]
[Total 10]

CT8 September 2012 Q3

3 Let $(X_t; t \geq 0)$ be a stochastic process satisfying $dX_t = \mu_t dt + \sigma_t\, dW_t$ where W_t is a standard Brownian motion.

Let $f(t,x)$ be a function, twice partially differentiable with respect to x, once with respect to t.

(i) State the stochastic differential equation for $f(t, X_t)$. [2]

Let $dX_t = \lambda X_t dt + \sigma dW_t$.

(ii) Solve this differential equation, by considering $X_t = U_t e^{\lambda t}$ or otherwise. [6]
[Total 8]

CT8 April 2010 Q1

4 Let $B(t,T)$ be the price at time t of a zero-coupon bond paying £1 at time T, r_t be the short-rate of interest, $\mathbb{P}$ be the real world probability measure and $\mathbb{Q}$ the risk neutral probability measure.

(i) Write down an equation for the price of a zero-coupon bond under the risk-neutral pricing approach. [2]

(ii) State the Stochastic Differential Equation (SDE) of the short rate r_t under Q for the Vasicek model and the general type of process this SDE represents. [3]

(iii) Solve the SDE for the short rate r_t from part (ii). [5]

(iv) Identify the form of the distribution of the zero-coupon bond price under this model. [2]

[Total 12]

CT8 April 2011 Q10

5 The effective risk free interest rate is 4% p.a. Company A has issued a one year zero coupon bond with a yield of 6% p.a. and Company B has issued a one year zero coupon bond with a yield of 8% p.a. All rates are annually compounded.

Recovery rates on the bonds in the event of default are zero and there are no frictional costs.

(i) Calculate the risk neutral implied default probability of each bond. [2]

(ii) Calculate the 95% VaR and 95% TailVaR at the end of the year for the following portfolios, assuming defaults by A and B are independent:

(a) £100 invested in A bonds.
(b) £100 invested in B bonds.
(c) £50 invested in A bonds and £50 invested in B bonds.

[6]

(iii) Comment on your answers to part (ii). [4]

[Total 12]

CT8 September 2012 Q1

6 (i) Describe the Merton model for pricing a defaultable bond. [4]

A very highly geared company, Risky plc, has issued zero coupon bonds payable in three years' time for a total nominal amount of £3,200m.

A Black-Scholes model for the value of the company is adopted.

(ii) Derive an expression for the value of the debt. [3]

The current gross value of the company is £6,979m. The continuously compounded risk-free interest rate is 2% p.a. and the price per £100 nominal of the bond is £92.603.

An insurance company is offering default insurance on Risky plc. They will charge a premium of £55,000 for a contract which pays £1m at the end of three years if Risky plc defaults.

(iii) Discuss, by determining the risk-neutral premium, whether this presents an arbitrage opportunity. You should state any assumptions you make. [4]

[Total 11]

CT8 September 2013 Q10

7 Claims on a portfolio of insurance policies arrive as a Poisson process with parameter 100. Individual claim amounts follow a normal distribution with mean 30 and variance 5^2. The insurer calculates premiums using a premium loading of 20% and has initial surplus of 100.

(i) Define carefully the ruin probabilities $\psi(100)$, $\psi(100,1)$ and $\psi_1(100,1)$. [3]

(ii) Define the adjustment coefficient *R*. [1]

For this portfolio the value of *R* is 0.011.

(iii) Calculate an upper bound for $\psi(100)$ and an estimate of $\psi_1(100,1)$. [5]

(iv) Comment on the results in part (iii). [2]

[Total 11]

CT6 April 2012 Q11 (modified)

8 A one-year European call option on a non-dividend paying stock in Company ABC has a strike of $150.

The continuously compounded risk-free rate is 2% p.a. The current stock price is $117.98. Assume that the market follows the assumptions of a Black-Scholes model.

An institutional investor holds a delta-hedged portfolio with 100,000 call options, no cash and short 18,673 shares of Company ABC.

(i) Calculate the delta of the call option. [2]

The implied volatility for the underlying is $\sigma = 22\%$

(ii) Calculate the price of a one-year put on the same stock with a strike of $150. [2]

The investor retains their holding of call options and trades in the put and the stock to achieve a delta and gamma-hedged portfolio.

(iii) Calculate the investor's new holdings of the put and the stock. [4]

(iv) Explain what it means for this portfolio to be delta and gamma hedged. [2]
[Total 10]

CT8 April 2013 Q9 (modified)

9 (i) State the assumptions underlying the Black-Scholes market. [3]

(ii) State the defining characteristics of Brownian Motion. [2]

(iii) Explain what an examination of past option prices tell us about the assumptions in parts (i) and (ii). [4]
[Total 9]

CT8 April 2012 Q1

10 Describe three behavioural biases, indicating, for each, how they can lead to non-rational expectations on the part of investors. [10]

END OF PAPER