INSTITUTE AND FACULTY OF ACTUARIES

Curriculum 2019

SPECIMEN EXAMINATION

Subject CM2B – Financial Engineering and Loss Reserving

Time allowed: One hour and forty five minutes

INSTRUCTIONS TO THE CANDIDATE

1. *You are given this question paper and three Excel files.*

2. *Mark allocations are shown in brackets. This exam has a total of 100 marks.*

3. *Attempt all three questions, each question is to be answered in a separate Excel file. There is a total of three Excel files, one for each question provided.*

4. *Your workings for each part of each question should be in a separate worksheet. For example, question 1(i) should be within sheet (i) of the spreadsheet named CM2 Q1.*

5. *Where possible, summarise your answers for each question in the worksheet named 'Answers'.*

6. *Upload the three Excel files as your solutions.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 A stock S_t is currently priced at £100. A put option on S_t with maturity in five years and a strike price of £80 is currently priced at £4.71. The risk-free force of interest is 4% pa. These parameters are provided in the spreadsheet named CM2 Q1.

(i) Find the implied volatility of S_t. [4]

A call option is also available on the same stock with the same term to maturity, but with a strike price of £150.

(ii) Calculate the value of the call option at time 0. [2]

The share price at time 5 is assumed to take a range of integer values with set probabilities, as listed in the spreadsheet model.

(iii) Calculate the put and call option payoffs for each of the integer values. [4]

An investor has £100 available to invest at time 0 in a combination of the stock and the two options. He invests in 0.5 units of the stock, 1 put option and 3.5 call options. The investor makes decisions based on a log utility function, so the utility of wealth *x* is

$$U(x) = \begin{cases} \ln(x) \text{ for } x>0, \\ 0 \text{ for } x<=0. \end{cases}$$

(iv)

(a) Calculate the value of the investor's portfolio at time 5 for each of the possible stock prices.

(b) Calculate the utility of the investor's portfolio at time 5 for each of the possible stock prices.

(c) Calculate the expected utility of the investor's portfolio at time 5 to four decimal places. [10]

The investor now decides not to hold any units of the stock, but instead invest the whole £100 only in the two options.

(v) Calculate, to four decimal places, the number of units of each option that the investor should hold to maximise his expected utility of wealth at time 5. [14]

(vi) Plot a chart, based on your solution to part (v), of the investor's utility at time 5 against the share price at that time. Comment on the shape of the chart and suggest how the investor could obtain a flatter utility curve. [6]

[Total 40]

2 Consider a three-period binomial model for a stock with current price is $S_0 = \$100$. This stock does not pay any dividend.

Suppose that:
- Over each of the next one-year periods, the stock price can either move up by a factor u=1.2 or down by d=0.8;
- The continuously compounded risk-free rate is 3% in period 1, 4% in period 2 and 5% in period 3.

These parameters are provided in the spreadsheet named CM2 Q2.

(i) Calculate the initial price of a European put option written on the stock with strike price K=90 and maturity three years. [10]

(ii) Find the price of a European call option written on the stock with strike price K=90 and maturity three years. (You should justify your answer). [6]

(iii) Find the price of an American put option written on the stock with strike price K=90 and maturity three years. [8]

[Total 24]

3 You are given daily stock prices for three firms from 3rd of January 2017 to 31st of March 2017. These stock prices are provided in the spreadsheet named CM2 Q3. Assume that on 3rd of January 2017 the risk-free force of interest for a three-month investment was 0.6% per annum. Assume that no dividends are paid.

(i) Calculate the forward price on 3rd January 2017 for a three-month forward contract on each of the three stocks. [6]

Assume that an investor takes, on 3rd of January 2017, one long position in a three-month forward contract of Oil, one long position in a three-month forward contract of Airline, and one short position in a three-month forward contract of Retailer.

(ii) Plot the unrealised profit (loss) for the investor on each day until the maturity of the contracts. [20]

Assume that on 1st April 2017 the risk-free force of interest fell to 0.5% per annum, and that on the same day, Retailer paid a cash dividend of amount D.

(iii) Calculate the amount of dividend "D" that gives a non-arbitrage price of 38 for a four-month forward contract on Retailer as at 3rd January 2017. [10]

[Total 36]

END OF PAPER