INSTITUTE AND FACULTY OF ACTUARIES

Curriculum 2019

SPECIMEN EXAMINATION

Subject CP2 – Actuarial Modelling

Paper Two

Time allowed: Three hours and fifteen minutes

INSTRUCTIONS TO THE CANDIDATE

1. *You must build your model from the beginning and not use an imported e-template.*

Your file names must include your ARN, the name of the document and the paper sat (e.g. 9000000-Summary-Paper1) and each file should contain your ARN as a header or footer.

Please note that the content of this booklet is confidential and students are not to discuss or reveal the contents under any circumstances nor are they to be used in a further attempt at the exam.

If you encounter any issues during the examination please contact the Online Education team at online_exams@actuaries.org.uk T. 0044 (0) 1865 268 255

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Exam requirements

1. Read the background document, which describes the scenarios that have been modelled and documented for this project and the work which remains outstanding. Technical assistance for the modelling work, should you require it, can be found in the additional guidance contained in this booklet.

No marks will be deducted for the use of this guidance.

2. Read the audit trail which has been written by your colleague, another actuarial student, for the calculations that they performed. This will assist you in following and understanding the calculations performed in the Excel model provided.

You are not required to add to or amend the audit trail.

3. Build on the model provided to produce the following additional calculations. You should ensure that the additional spreadsheet work which you perform contains appropriate self-checks.

(i) Determine the frequency distribution of the simulated aggregate gross (i.e. before reinsurance) claim amounts using a step size of £500,000 (i.e. the number of aggregate claim amounts falling within each £500,000 band). [3]

(ii) Calculate the amounts that would be paid by the insurance company and the amounts that would be paid from the reinsurer under the alternative reinsurance, for each simulation. [3]

(iii) Calculate the following for both the current and alternative reinsurance:

(a) the required summary statistics
(b) the probability of receiving any reinsurance recovery [3]

You should assume that your colleague's calculations have been checked and are correct.

4. Illustrate in your model the following using two suitable charts:

- the distribution of simulated aggregate claim amounts, as determined in 3(i)
- a comparison of the four summary statistics for the current and alternative reinsurance arrangements, as determined in 3(iii)(a)

[6]

[Sub-total 15]

5. Prepare a summary document of around five to seven pages, capturing the main features and results of the work done by you and your colleague. You can assume that the summary is being prepared for your boss, a senior actuary, who will present the work to the Chief Executive Officer.

Your summary should include the following:

- purpose of the project, data, method and assumptions used by you and your colleague
- results, including charts
- commentary on the results, key conclusions and suggested next steps

The summary should cover the full scope of the project, including the modelling already done by your colleague in the spreadsheet provided.

You are not required to add to or amend the audit trail.

Marks available for the summary:

Methodology (including purpose, data, method and assumptions):	**[20]**
Results, including charts	**[5]**
Commentary on results and conclusions	**[25]**
Next steps	**[25]**
Drafting	**[10]**

[Sub-total 85]

[Total 100]

Background

In the country of Actuaria, the TransQuick Insurance Company (TQIC) insures a large proportion of the country's transportation industry. You are an actuarial student working for TQIC.

Most claims that arise on TQIC's transportation insurance portfolio relate to theft of goods or damage to lorries. However, Actuaria suffers occasional hurricanes which can result in significant claims in relation to damage to property, such as warehouses.

TQIC is currently looking at renewing its reinsurance arrangement for next year. The reinsurance company that currently reinsures TQIC's transportation insurance portfolio has offered an alternative reinsurance arrangement, quoting the same cost as the current arrangement.

The current and alternative reinsurance arrangements are as follows (amounts stated in Actuarian Pounds £):

Current reinsurance: retention limit £1,500,000, no upper limit
Alternative reinsurance: retention limit £1,250,000, upper limit £2,000,000

Under these two arrangements:

- The total individual claims incurred over the year on the portfolio are added together to produce the "aggregate claim amount" (S).

- If S is below the retention limit, then TQIC does not receive any payment from the reinsurer.

Under the current reinsurance arrangement:

- If S is above the retention limit (X), then TQIC receives a payment of $S–X$ from the reinsurer.

Under the alternative reinsurance arrangement:

- If S is above the retention limit (X) and below the upper limit (Y), then TQIC receives a payment of $S–X$ from the reinsurer.

- If S is above Y, then TQIC receives a payment of $Y–X$ from the reinsurer.

Your boss, a senior actuary, has asked you to complete a modelling exercise to help determine which of the two reinsurance arrangements TQIC should purchase.

One of your colleagues has already built a simulation model to illustrate 100 possible aggregate claim amount outcomes from TQIC's transportation insurance portfolio, over the next year.

Based on past data, she has estimated that the number of claim events that could occur over the year follows a Poisson distribution with parameter 20. She generated 100 randomly simulated numbers from this distribution, which lie within the range 0 to 40.

Your colleague has also estimated a distribution for the individual claim amounts, i.e. the amount that would have to be paid out by TQIC in respect of an individual claim before allowing for any reinsurance:

Claim amount range (£)	*Average claim amount x (£)*	*Probability of claim amount x p(x)*
0 – 25,000	14,000	0.225
25,000 – 50,000	38,000	0.425
50,000 – 100,000	80,000	0.250
100,000 – 500,000	275,000	0.075
> 500,000	1,000,000	0.025

She has used the above claim amount distribution, together with a set of random variables generated from a continuous Uniform distribution on [0,1], to simulate a large number of random individual claim amounts.

She has then combined these individual claim amount simulations with the simulated numbers of claims from the Poisson distribution to produce a set of 100 aggregate claim amount simulations (before reinsurance is taken into account).

Your boss requires calculation of the amount that would be paid by TQIC and the amount that would be paid by the reinsurer, for each simulation and for each arrangement, and calculation of a number of summary statistics for both arrangements.

Your colleague has already calculated the retained amounts and the recoveries due under the current arrangement. However, she is no longer able to work on this project and has handed over to you the spreadsheet which contains what she has produced so far. Your boss has asked you to complete the required calculations using this spreadsheet.

He has therefore first asked you to calculate the amount that would be paid by TQIC and the amount that would be paid by the reinsurer for the alternative arrangement.

He has then asked you to produce the following summary statistics across the simulations, separately for each arrangement:

- minimum reinsurance recovery
- maximum reinsurance recovery
- mean reinsurance recovery
- median reinsurance recovery

He would also like you to calculate, for each arrangement, the simulated probability of receiving any reinsurance recovery.

He would like your additional modelling to include appropriate checks where necessary.

Based on these calculations, he has also asked for your recommendation as to which would be the more appropriate arrangement for TQIC to purchase.

Your boss needs you to prepare a summary document covering both the current and alternative arrangements.

He is out of the office visiting a client and cannot be contacted for the next three hours. He would like the additional calculations finished and the summary document written up ready for his return, when he will review them and present your findings to the Chief Executive Officer.

Your colleague has produced an audit trail of her model. A copy of the audit trail is contained in this booklet and an electronic copy of the model is provided.

You are not expected to include the additional modelling request in the audit trail, but your results should be included in the summary.

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Audit trail

The following audit trail should be read alongside the provided model.

Purpose of the Model

The purpose of the model is to complete the following calculations:

- Use a set of $U[0,1]$ random numbers to simulate a set of individual claim amounts incurred by an insurance portfolio, based on the average amount for each band of possible claim amounts.

- Combine these simulated individual claim amounts with the Poisson simulated numbers (which represent the number of events occurring during the year) to obtain 100 simulations of the aggregate claim amount over the forthcoming year.

- Calculate the retention and recovery under each simulation for the current reinsurance arrangement.

***Data* worksheet**

This worksheet includes 100 random numbers generated from a Poisson distribution and $100 \times 50 = 5{,}000$ random numbers generated from a continuous Uniform distribution on [0,1].

***Simulation* worksheet**

In this worksheet the random numbers from "Data" are used to simulate individual claim amounts and hence 100 outcomes of the aggregate claim amount over the forthcoming year.

Assumptions

- The average claim amounts and probabilities estimated for modelling individual claim amounts (entered into cells A8:B12) are appropriate.

- Claim frequencies follow the Poisson distribution as specified.

- No allowance for inflation is required.

The simulated aggregate claim amounts (S) are determined as follows:

- For each claim band with average claim amount x, the cumulative distribution function $P(x)$ is calculated as the sum of $p(y)$ over all y less than or equal to x. See cells C7:C12. The claim amounts to simulate are brought through in cells D7:D11.

- In cells M8:AZ107 (a 100×50 table consistent with the format of the random numbers), the $U[0,1]$ random numbers are mapped to the average claim amounts x. This involves comparing each random number r with $P(x)$. The lowest x for which $P(x) > r$ is then taken as the simulated individual claim amount. This is achieved in the spreadsheet using a VLOOKUP table, which looks up each r against the values of $P(x)$ and returns the first x for which $r < P(x)$, as required.

- To the left of this table, column L includes the 100 Poisson random variables from the "Data" worksheet.

- Within the table, the simulated individual claim amounts are set to zero for all columns that are more than f columns across the table, where f is the raw data Poisson random number for that particular row. This results in there being exactly f non-zero simulated individual claim amounts in each row of the 100×50 table. These represent the expected f individual claims simulated to occur during the following year.

- Column I calculates the total of these individual simulated claim amounts for each row, and hence represents the aggregate claim amount (S) for each simulation.

The simulations are checked as follows:

- In columns G and H, the count of non-zero individual claim amounts is determined for each simulation and checked against the Poisson number f for that simulation.

- Above the table in cells R3:S4, the following is calculated:

  - the mean simulated aggregate claim amount

  - the expected aggregate claim amount (which is equal to the expected number of events (Poisson parameter) × expected average individual amount (determined as average claim amount (x) multiplied by $p(x)$, summed over all bands of x)

- The expected figure is reasonably close to the simulated average.

***Reinsurance* worksheet**

This worksheet calculates, for each simulation, the retention and recovery for the current reinsurance arrangement covering the insurance portfolio.

Assumptions

- All claim events are covered by the reinsurance arrangement.

In cells B6:B7, the characteristics of the current reinsurance arrangement are entered: i.e. the retention limit (X) and the upper limit (Y). [Note: this reinsurance arrangement has no upper limit so for ease of calculation this has been input as being an extremely high upper limit. This will not affect the results as it is set to be high enough not to be breached in any simulation.]

Column B brings in the aggregate claim amount S for each simulation, from the Simulation worksheet.

For each simulation, the insurer's retention is calculated in column D as:

- If the claim S is less than the retention limit X, the whole claim S is retained.

- If the claim S is greater than the retention limit X, then amount X is retained.

- If the claim S exceeds the upper limit Y, then the excess of S over Y is also retained by the insurer.

  So Retention = Min { S, X } + Max { $S - Y$, 0 }.

For each simulation, the insurer's recovery from the reinsurer is calculated in column E as:

- If the claim S is less than the retention limit X, nothing is recovered from the reinsurer.
- If the claim S is greater than the retention limit X, then up to $S - X$ may be recovered.
- However, if the claim S exceeds the upper limit Y then the maximum that can be recovered is $Y - X$

  So Reinsurance Recovery = Min [Max { $S - X$, 0 } , $Y - X$].

There is a check in column F to show Retention + Recovery = Total aggregate claim amount.

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