

# **INSTITUTE AND FACULTY OF ACTUARIES**

## **SUMMARY**

Specimen 2019

**CP2: Actuarial Modelling**

**Paper 2**

## TQIC Reinsurance Renewal

### Objective

The objective of this project is to use random numbers to simulate a set of 100 aggregate claim amounts incurred during the forthcoming year, under TransQuick Insurance Company's (TQIC) portfolio of transportation insurance policies.

These simulations are then used to analyse the retentions and recoveries that would be achieved under two different reinsurance arrangements:

- Reinsurance 1 – the current arrangement; and
- Reinsurance 2 – the alternative arrangement proposed

The overall objective is to use the model to determine which might be the more appropriate reinsurance arrangement to purchase for next year.

### Data

The data provided comprises a set of 100 numbers randomly generated from a Poisson distribution with parameter 20, and a set of 5,000 numbers randomly generated from a continuous Uniform distribution on [0,1].

It is assumed that this data has been validated, including checking that the means and variances of the random number samples are close to those expected.

Information has been provided on the estimated probabilities of individual claim events being within certain bands and an estimated average claim amount for each band.

Details of the retention limits and upper limits have also been given:

Reinsurance 1 – current: retention limit £1,500,000, no upper limit  
Reinsurance 2 – alternative: retention limit £1,250,000, upper limit £2,000,000

### Assumptions

- The average claim amounts and probabilities provided for modelling individual claim amounts are appropriate and complete.
- Claim frequencies follow the Poisson distribution as specified.
- No allowance for inflation is required **i.e. the claim amount figures provided have already been inflation-adjusted to an appropriate level for the forthcoming year, or alternatively inflation is low enough to be ignored.**
- **Individual claim amounts are independent, and so correlations between the amounts paid under claim events arising during the year can be ignored**

- All claim events are covered by the reinsurance arrangements.
- **The reinsurance company does not default on the payment of any recoveries.**
- **There is no delay between TQIC incurring a claim and recovering the amount from the reinsurer i.e. no reporting delays are modelled.**

## Method

### Simulations

The set of  $100 \times 40$  Uniform random numbers between 0 and 1 has been used to simulate individual claim amounts, where these are based on the average claim amount for each band as set out in the table below:

<i>Claim amount range (£)</i>	<i>Average claim amount <math>x</math> (£)</i>	<i>Probability of claim amount <math>x</math> <math>p(x)</math></i>
0 – 25,000	14,000	0.225
25,000 – 50,000	38,000	0.425
50,000 – 100,000	80,000	0.250
100,000 – 500,000	275,000	0.075
> 500,000	1,000,000	0.025

The individual claim amount simulations have been performed using the fact that the cumulative distribution function  $P(x)$  (the sum of the probability function  $p(y)$  over all average claim amounts  $y$  that are less than or equal to  $x$ ) must also lie between 0 and 1, and by mapping the random numbers onto  $P(x)$  to obtain  $x$ .

For each of the 100 simulations, only the first  $f$  of the 40 simulated individual claim amounts is counted, where  $f$  is the claim event frequency taken from the Poisson random variable data (a different  $f$  being used for each simulation). The other simulated individual claim amounts are set to zero.

The  $f$  simulated individual claim amounts are summed for each simulation, giving a set of 100 aggregate claim amounts ( $S$ ). These are summarised into frequency bands with step size 500,000.

The overall average aggregate claim amount has been calculated and compared against the expected aggregate claim amount, which is determined as the expected average number of claims (i.e. the mean of the Poisson distribution, 20) multiplied by the expected average individual claim amount (calculated as the average claim amount ( $x$ ) multiplied by  $p(x)$ , summed over all  $x$ ). The expected amount is reasonably close to the simulated average amount, which provides confidence in the simulations.

### **Reinsurance calculations**

The following two reinsurance arrangements (having the same cost) have been investigated:

Reinsurance 1 – current: retention limit £1,500,000, no upper limit.

Reinsurance 2 – alternative: retention limit £1,250,000, upper limit £2,000,000

For each reinsurance, and for each simulation, the insurer retention and reinsurance recovery has been calculated using the following formulae:

$$\text{TQIC retention} = \text{Min} \{ S, X \} + \text{Max} \{ S - Y, 0 \}$$

$$\text{TQIC reinsurance recovery} = \text{Min} [ \text{Max} \{ S - X, 0 \}, Y - X ]$$

where  $S$  = aggregate claim amount,  $X$  = retention limit,  $Y$  = upper limit.

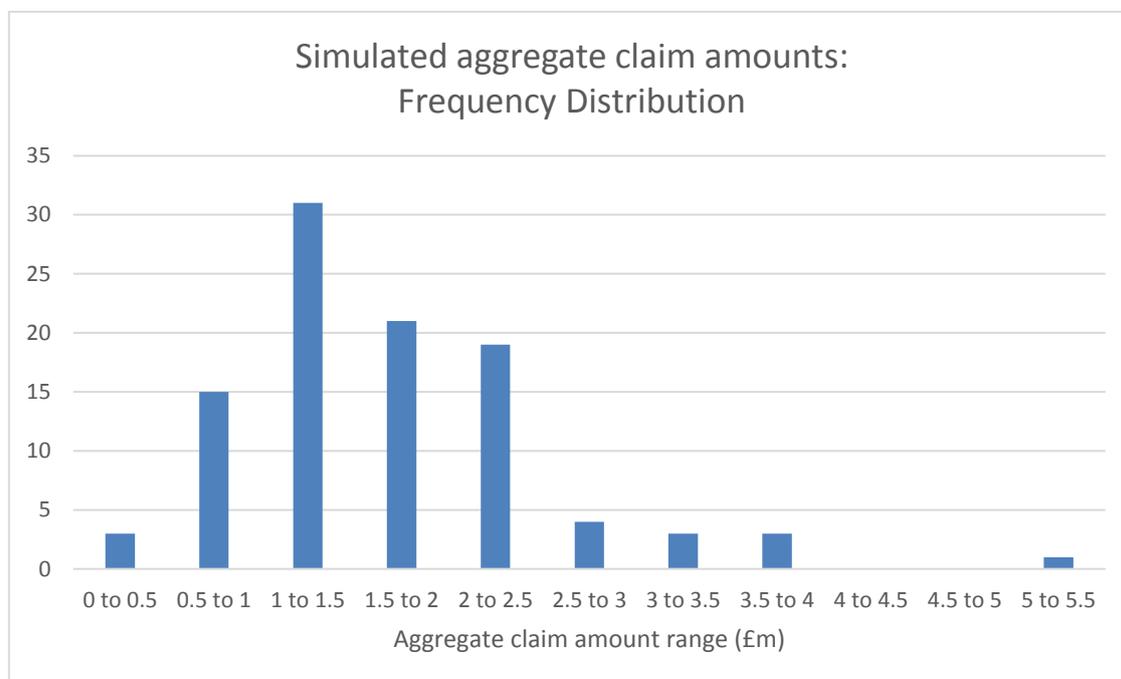
Summary statistics have also been calculated for each reinsurance: minimum, maximum, mean and median insurance recoveries across all simulations.

The simulated probability of the insurer receiving any reinsurance recovery has also been calculated, as the number of simulations leading to any recovery divided by the total number of simulations.

## Results

### Aggregate claims distribution

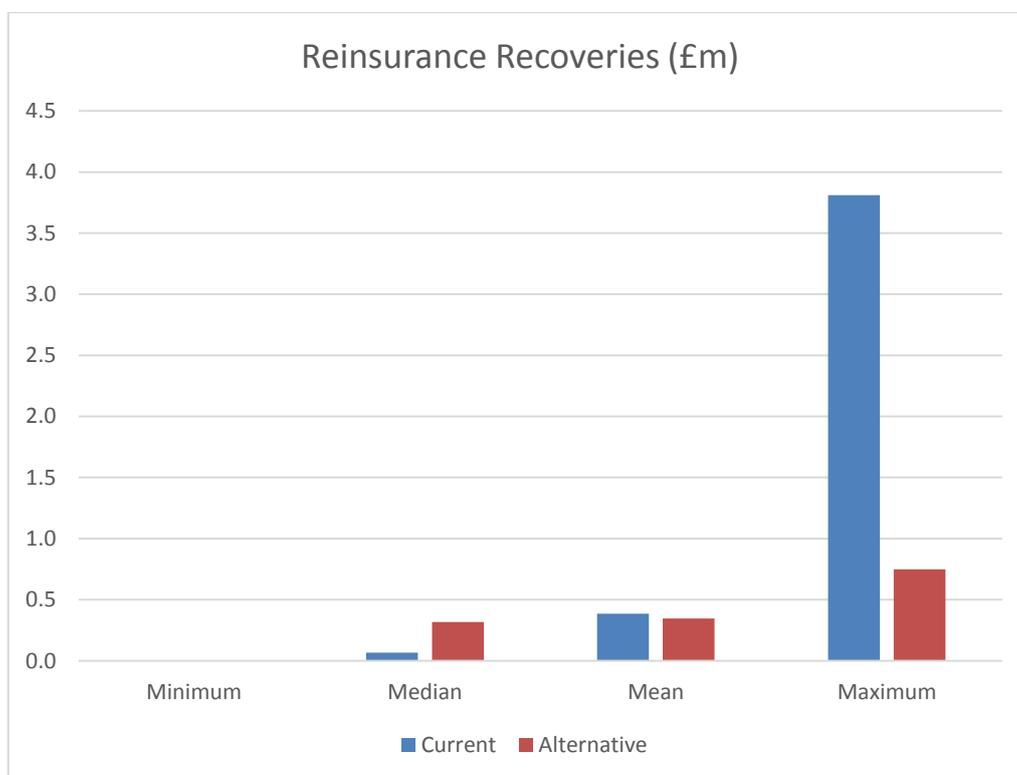
The bar chart below shows the aggregate claims amount distribution. A step size of 500,000 has been used for the distribution.



- The bar chart shows that the distribution of simulated aggregate claim amounts is broadly Normal but with a skew towards the higher amounts. This is as expected, given the law of large numbers and the occasional occurrence of very high individual claims.
- It would be worth investigating the underlying losses in the one simulation that has generated an aggregate claim amount of between £5m and £5.5m. This looks to be an outlier in the model and may challenge the assumption that the underlying data is correct.

## Reinsurance 1 vs Reinsurance 2

The bar chart below shows summary statistics for the recovery amounts under the current reinsurance arrangement and the alternative reinsurance arrangement proposed.



The figures in the graph above are also included here for completeness, as well giving the probability of any recovery under both arrangements:

	<i>Reinsurance 1</i>	<i>Reinsurance 2</i>
Minimum	£0	£0
Median	£0.07m	£0.32m
Mean	£0.38m	£0.35m
Maximum	£3.81m	£0.75m
Prob of recovery	0.51	0.64

- The bar chart above shows that the minimum recovery for both reinsurances is zero, which reflects some of the simulated aggregate claim amounts being less than the retention limits.
- The maximum recovery is much higher for Reinsurance 1, which is as expected since there is no upper limit for that reinsurance (and the maximum recovery for Reinsurance 2 is fixed at the difference between the upper limit and the retention limit).
- The means are similar, which is consistent with the two arrangements having similar cost.

- However, the median is lower for Reinsurance 1 than for Reinsurance 2. This reflects the higher retention limit for Reinsurance 1, which means that there will be a larger number of zero recoveries.
- The probability of recovery is higher for Reinsurance 2 than for Reinsurance 1, which is as expected since the retention limit is lower for Reinsurance 2.

## **Conclusions**

The median simulated recoveries are higher for Reinsurance 2 than for Reinsurance 1 and the means are quite close. The probability of making a recovery is also higher for Reinsurance 2. This might suggest that this would be the better arrangement to enter into, if both have the same cost to TQIC.

However, Reinsurance 1 does not have an upper limit and for very high aggregate claim amounts the recovery from the reinsurer is significantly higher than under Reinsurance 2, which does have an upper limit. This means that TQIC would be much more exposed to high aggregate losses if Reinsurance 2 is chosen.

One simulation in the model illustrates the potential of a large event with a gross loss in excess of £5m. If the underlying assumptions causing this outlying simulation are reasonable then this would also suggest that Reinsurance 1 might represent better protection at the top of the distribution.

Which reinsurance is more appropriate for the insurer will depend on the company's risk appetite and the amount of capital available to withstand very significant aggregate losses.

If the risk appetite is relatively conservative and there is limited capital available, then the insurance company should choose Reinsurance 1.

## **Next steps**

- Repeat the calculations using a higher number of random numbers and simulations.
- Validate the information provided, for example use other data sources such as industry data from other similar insurance companies; in particular, compare individual claim amount averages.
- Test the sensitivity of the results to other variables, such as the number of claim events (Poisson parameter) and the probabilities assigned to each individual claim amount band.
- Test the impact of more extreme scenarios than those included within these simulations, e.g. a "1 in 200 year" storm.
- Allow for correlations between claim events, for example to reflect the fact that a storm may result in a high concentration of large claim amounts.

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- Verify whether inflation has been allowed for in the figures provided, and if not then allow for it in the modelling.
- Assess whether climate change might influence the results of the model especially for the highest assumed average loss of £1m in the 'greater than £500,000' band.
- Use a catastrophe model in order to model catastrophic losses. This is an alternative model to using historic claims data, where cat losses may be understated, or overstated in the data.
- Use a step size of 250,000 to graph the aggregate claims distribution. This will allow better analysis of Reinsurance 2 with the lower retention.
- Use different underlying statistical distributions for the claim frequency and individual amount variables.
- Adjust the figures to allow for any changes in the underlying portfolio in the forthcoming year, as compared with the portfolio on which the frequency and claims amount information is based.
- Assess whether there are other influences that might result in future claims differing from historic claims, and allow for them.
- Check the financial strength or credit rating of the new reinsurer, and confirm that TQIC are comfortable with it.
- Allow for reinsurer defaults in the model by applying default probabilities to the reinsurer claim amounts.
- Investigate whether any other reinsurance arrangements are available for a similar cost, e.g. from other reinsurers, ideally with an upper limit.
- Discuss with the insurance company the amount of variability in net (i.e. post reinsurance) insurance losses that could be withstood. For example, could compare the highest simulated net losses against the amount of capital available in the insurance company, particularly for Reinsurance 2.
- Obtain a peer review of the work performed.

**END OF SUMMARY**